5 Implicit Differentiation

Learning Objectives: After completing this section, we should be able to

• find the derivative of a function when it is implicitly defined.

5.1 Chain Rule Review

Chain Rule Problem from a Table:

Example. We have f'(1) = 2, g(0) = 1, and g'(0) = 3. Compute $\frac{d}{dx} (f(x + g(x)))$ at x = 0.

You try!

Example. Let $f(x) = x^2 + 2(g(x))^3$. Find f'(x).

5.2 Implicit Equations

An **explicit equation** is

An **implicit equation** is

Even though we don't know y = f(x) explicitly,

Example. $x^3 + y^3 - 9xy = 0$. Find $\frac{dy}{dx}$.

Example Continued.

Example Continued. What does this $\frac{dy}{dx}$ tell us?

Example. Suppose $y^2 e^x + x e^{y^2} = 4 + x$. Find $\frac{dy}{dx}$.

You try!

Example. Suppose $\tan(x^2y) = x + y^3 \sin(x)$. Find $\frac{dy}{dx}$.

Example. The energy of a capacitor is given by $E = \frac{1}{2} \frac{Q^2}{C}$.